ELSEVIER Journal of Constructional Steel Research 58 (2002) 995–1013

JOURNAL OF
CONSTRUCTIONAL
STEEL RESEARCH

www.elsevier.com/locate/jcsr

Refined 3D finite element modeling of partially-restrained connections including slip

A.M. Citipitioglu, R.M. Haj-Ali *, D.W. White

School of Civil and Environmental Engineering, Structural Engineering and Mechanics Group, 790 Atlantic Drive, Georgia Institute of Technology, Atlanta, GA 30332-0355, USA

Received 16 May 2001; received in revised form 16 August 2001; accepted 9 October 2001

Abstract

This study presents an approach for refined parametric three-dimensional (3D) analysis of partially-restrained (PR) bolted steel beam-column connections. The models include the effects of slip by utilizing a general contact scheme. Non-linear 3D continuum elements are used for all parts of the connection and the contact conditions between all the components are explicitly recognized. A method for applying pretension in the bolts is introduced and verified. The effect of several geometrical and material parameters on the overall moment–rotation response of two connection configurations subject to static loading is studied. Models with parameters drawn from a previous experimental study of top and bottom seat angle connections are generated in order to compare the analyses with test results, with good prediction shown by the 3D refined models. The proposed 3D modeling approach is general and can be applied for accurate modeling of a wide range of other types of PR connections. A pronounced effect of slip and friction, between the connection components is shown with connections having thicker (stiffer) seat angles. This study demonstrates the effects of clamping through the bolts and contact between the components on the overall non-linear moment–rotation response. Equivalent moment–rotation responses of pull-test simulations are compared to FE model responses of full connections without web angles. The moment–rotation from the pull test is shown to be equivalent to that of the full FE model for small rotations. As the rotation increases a softer response is shown by the pull tests. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Connection; Partially-restrained; Semi-rigid; Bolted connection; Beam-column; Steel; Pretension; Bolt slip; Three-dimensional; Non-linear; Finite element

---

* Corresponding author. Tel.: +1-404-894-4716; fax: +1-404-894-0211.
E-mail address: rami.haj-ali@ce.gatech.edu (R.M. Haj-Ali).

PII: S0143-974X(01)00087-6

1. Introduction

The effect of partially-restrained (PR) connections on the behavior of steel frames and their potential economical benefits is well recognized [18]. However, many structural analysis and design approaches still consider connections as either fixed or pinned. This assumption is mainly due to convenience and the lack of common analysis and design approaches that address PR connections. Despite many full-scale experimental studies that have been conducted to date, there is still a need for a better understanding of the mechanisms that effect the non-linear behavior of PR connections [8].

Non-linear moment rotation response of connections was recognized in the early 1930s. Standardized functions have been developed starting from basic linear and bilinear approximations to more sophisticated models based on polynomials, cubic B-splines and power functions fitted to available experimental data. Sherbourne and Bahaari [13] have recently presented a review of these functions. Frye and Morris [9] were among the first to incorporate these standardized moment–rotation functions in steel plane frame analysis to investigate the effect of the connections on the frame behavior.

Moment rotation functions can be useful for designers in practice. These usually include small number of parameters taken into account from limited test data. The lack of a large and parametrized experimental database does not allow for generating standardized functions. Thus, there is a need to be able to analytically generate a reliable moment–rotation response of PR connections that can be used in analysis and design.

Non-linear finite elements are an attractive tool for modeling connections. Early attempts to use finite elements for analysis of PR connections was by Krishnamurthy [11]. As in many early studies using finite elements, many simplifications are made due to the limitations of computational power. More recent studies using finite elements in modeling connections have focused on end plate connections [6,7,10,13,14]. In these studies 2D and 3D models are used with various simplifications in the geometry of members, the bolts, and contact conditions. The effect of friction on the response of end plate connections is usually neglected in these models [7,10].

Azizinamini [3–5] preformed an extensive and detailed experimental study for top and bottom seat angle connections with double web angles along with pull tests. In addition, simplified 3D FE models for the pull tests are also studied. One quarter of the top angle in the connection is modeled with 3D elements to simulate a pull test. The force–displacement relation was converted to a moment–rotation relation in order to examine the role of the top angle on the behavior of the connection and approximate the overall response of the connection with a pull test. Different assumptions and simplifications are made in order to avoid detailed modeling and reduce the computational effort.

Yang et al. [19] consider a double web angle connection where the angles are bolted to the column flanges and welded to the beam web. The bolts and angle are modeled using 3D finite elements and wedge elements are used to model the weld region. Contact is included between the bolt head and angle. However, the contact between the bolt shank and hole is ignored.

In the studies on end plate connections and the double web angle connection, the bolts are transferring the loads axially, thus eliminating the need for combined contact and friction modeling between the bolts and members. These models are therefore limited to these types of PR connections. The bolted connections tested by Azizinamini et al. [4,5] are investigated in this study. These top–bottom bolted seat angle connections transfer the forces by friction by clamping the parts together with the bolts. Modeling such a mechanism requires the inclusion of contact and slip between the connection members.

In this study, a refined 3D modeling of PR bolted connections are performed recognizing contact and friction effects. The modeling approach is general and capable of modeling various types of geometries of PR connections by using parametric meshing techniques. Therefore the time of generating detailed 3D geometries is almost eliminated. A calibration method for the pretension of the bolts is presented. In this method, parametric solutions are first generated separately for a single bolt clamping semi-infinite plates. These solutions are used to specify initial pretension values for the bolts in the full connection. The correct pretension values are then examined and corrected in the full connection model to achieve accurate final values. It is shown in this study that the response of the bolted PR connections are sensitive to the pretension of the bolts, thus correctly modeling the pretension and slip is important.

2. Detailed modeling approach

Displacement-based 3D finite element (FE) models are used to predict the behavior of bolted PR connections. The geometry and mesh is established through a parametric mesh generator program. The ABAQUS [1] FE code is used to carry out the 3D FE analysis. Several parametric studies are performed in which experimental data of Azizinamini [4] is used and a large number of analyses are performed. The parametric investigation demonstrates the capability of the FE models to efficiently generate connection responses beyond experimental data.

Each part in the connection is formed and assembled using the TrueGrid [15] mesh generating software. The processes of creating a mesh and geometry are separated within this software. Making a part is analogous to sculpting. A meshed block is defined with different parameters. Excess volumes of certain parts of the block are removed to match the general shape of the part. The exposed areas of the remaining parts are projected to the exact surface geometry that is defined separately. Both the meshed block and the surface geometry are parametric allowing general modeling capability. Important features such as holes, fillets, surface definitions for contact and node definitions for boundary conditions are also included in the part definition which follows new parametric changes.

Different classes of structural shapes can be generated using the programming language of TrueGrid. A program library of parametric structural shapes and bolts is generated in this study. These programs are executed within TrueGrid to generate the specific components of the connection configuration and assemble these components to form the connection model. This versatility of this approach allows for a wide range of parametric studies to be conducted without time-consuming preprocessing.

3. Previous experimental work

The test setup of Azizinamini [4] is illustrated in Fig. 1. It consists of a pair of beams connected to a central stub column via top and seat angles bolted to the flanges of the beam and column. The double web angles are bolted both to the beam web and column flanges. High strength bolts and nuts, ASTM A325 heavy hex, are used with A325 hardened washers. The end of the beams is pinned while an actuator loads the central stub column.

Azizinamini computed the moment–rotation responses by using the force–displacement data acquired from the tests. The objective of these tests is to investigate

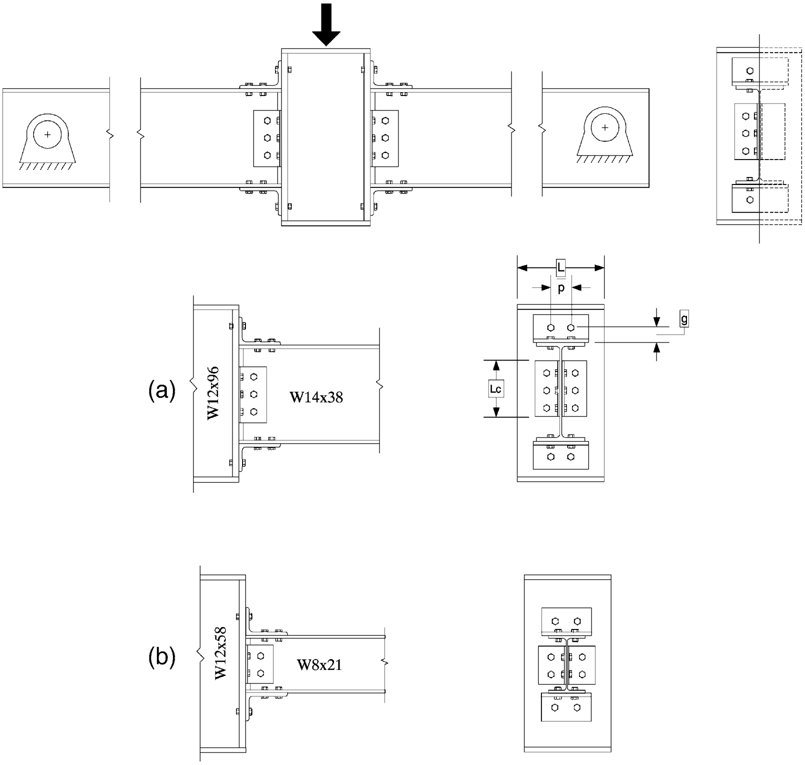

Fig. 1. Schematic representation of the connection test set-up used by Azizinamini et al. (1985, 1989); (a) 14S and (b) 8S type configuration.

the effects of different geometric parameters. These parameters include the angle thickness (t), angle lengths (L and L_C), bolt size, spacing (p) and the bolt gage on the column leg of the connection angles (g). Two test configurations are used; W14 × 38 and W8 × 21 beams are tested with W12 × 96 and W12 × 58 column-stub, respectively. Double web angles are connected with three bolts in the first test setup (W14 × 38beam). The second test setup (W8 × 21beam) includes two bolts through the double web angles. The column used in the experiments has relatively thick flanges and little or no plastic deformation is observed in the column and beams. Hence, the column and beams are reused throughout the tests. The test configurations are described in Table 1.

4. Finite element models

Three-dimensional refined FE models are proposed to generate the effective moment–rotation response of the PR connections. The experimental work and test results of Azizinamini [4] are modeled for this purpose. This is done in order to critically examine the ability of the proposed 3D models to capture the overall experimental response of the connections.

A representative 3D FE model of a top and bottom seat angle connection with double web angles is shown in Fig. 2. Half of the connection is modeled by using symmetry about the plane of the web. Only the flange of the column is modeled assuming that it is a sufficiently rigid part due to the stiffeners used of the column. The hex bolt heads are modeled as cylinders, taking into account the washers by averaging the diameter. The bolt holes are modeled as 0.3175 cm. (1/8 in.) larger than the bolt shaft diameter. It should be noted that increase in the bolt hole diameter is an assumed value that was not verified from the experimental studies.

The connection model is discretized using C3D8I eight-node brick elements with full integration and incompatible modes [1]. The performance of this continuum element has been compared with other formulations by Bursi and Jaspart [7] and has shown to give better results for bending-dominated problems with relatively small thickness. C3D6 six-node wedge elements are also used to model the core of the bolts. Wanzek and Gebbeken [17] stress the importance of through-thickness deformation in their analysis, therefore three elements through the thickness in the beam and columns are used to better capture the deformation behavior. The connection model includes five structural components and twelve bolts in the case of the 14Sx type test configurations. A total of 17,762 3D continuum elements are used in the 14Sx type models. In the case of the 8Sx models the five structural components and ten bolts compose of 16,296 3D elements.

Contact between all parts is explicitly modeled. The contact areas are the bolt shank-to-bolt holes and bolt head-to-components. The bolts clamp the components together in order to resist the applied rotation. This mechanism has a major effect on the performance of the connection and its response. The contact surfaces are defined and paired through the mesh generator program including areas anticipated to be in contact due to sliding. The general contact formulation used in ABAQUS

Table 1
Schedule of test specimens (Azizinamini [4])

Specimen number	Bolt size (in. dia.)	Beam section	Top and bottom flange angles				Web angles	
			Angle	Length (in.)	Gauge in leg on column flange *g (in.)*	Bolt spacing on column flange *p* (in.)	Angle	Length (in.)
14S1	3/4	W14×38	L6×4×3/8	8	2−1/2	5−1/2	2L4×3−1/2×1/4	8−1/2
14S2	3/4	W14×38	L6×4×1/2	8	2−1/2	5−1/2	2L4×3−1/2×1/4	8−1/2
14S3	3/4	W14×38	L6×4×3/8	8	2−1/2	5−1/2	2L4×3−1/2×1/4	5−1/2[a]
14S4	3/4	W14×38	L6×4×3/8	8	2−1/2	5−1/2	2L4×3−1/2×3/8	8−1/2
14S5	7/8	W14×38	L6×4×3/8	8	2−1/2	5−1/2	2L4×3−1/2×1/4	8−1/2
14S6	7/8	W14×38	L6×4×1/2	8	2−1/2	5−1/2	2L4×3−1/2×1/4	8−1/2
14S8	7/8	W14×38	L6×4×5/8	8	2−1/2	5−1/2	2L4×3−1/2×1/4	8−1/2
8S1	3/4	W8×21	L6×3−1/2×5/16	6	2	3−1/2	2L4×3−1/2×1/4	5−1/2
8S2	3/4	W8×21	L6×3−1/2×8/8	6	2	3−1/2	2L4×3−1/2×1/4	5−1/2
8S3	3/4	W8×21	L6×3−1/2×5/16	8	2	3−1/2	2L4×3−1/2×1/4	5−1/2
8S4	3/4	W8×21	L6×6×3/8	6	4−1/2	3−1/2	2L4×3−1/2×1/4	5−1/2
8S5	3/4	W8×21	L6×4×3/8	8	2−1/2	5−1/2	2L4×3−1/2×1/4	5−1/2
8S6	3/4	W8×21	L6×4×5/16	6	2−1/2	3−1/2	2L4×3−1/2×1/4	5−1/2
8S7	3/4	W8×21	L6×4×3/8	6	2−1/2	3−1/2	2L4×3−1/2×1/4	5−1/2
8S8	7/8	W8×21	L6×3−1/2×5/16	6	2	3−1/2	2L4×3−1/2×1/4	5−1/2
8S9	7/8	W8×21	L6×3−1/2×3/8	6	2	3−1/2	2L4×3−1/2×1/4	5−1/2
8S10	7/8	W8×21	L6×3−1/2×1/2	6	2	3−1/2	2L4×3−1/2×1/4	5−1/2

[a] Two bolts at 3 inch spacing, mounted on top two holes on stub column.

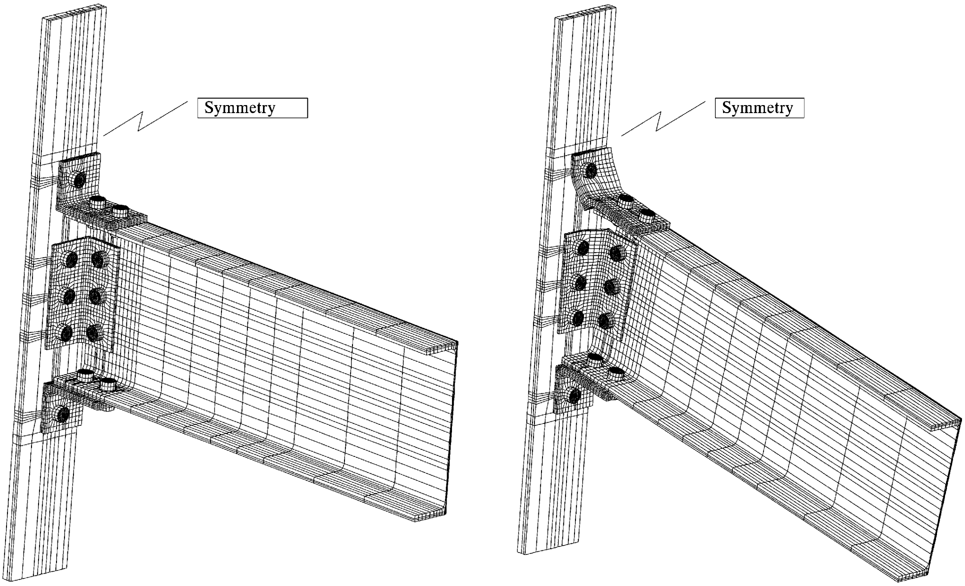

Fig. 2. 3D finite element partially-restrained connection model, deformed and undeformed shape.

involves a “master-slave” type algorithm [1]. This formulation recognizes the surfaces that are in contact or interpenetrate or slip and imposes constraints on the nodes of the slave surface such that they do not penetrate the master surface.

The pretension of the bolts and friction are critical parameters in these bolted connections. The forces are transferred through friction due to clamping between the members caused by the pretensioning of the bolts. No information is found on the amount of pretension applied to the bolts during the experimental study. Also the condition of the faying surfaces is unknown. Azizinamini [4] reported tightening the bolts with an air wrench using the turn-of-the-nut method. Therefore approximate common design values are used for modeling the pretension: 133 kN (30 kips) for the 19.1 mm (3/4 in.) and 178 kN (40 kips) for the 22.3 mm (7/8 in.) diameter A325 bolts. The friction coefficient of 0.33 for Class A surface is used [2,12,16].

The pretensioning of the bolts is required to achieve clamping between the parts for the connection. The pretension in the model is achieved in two steps. The first step employs bolts with shorter length shafts compared to the total thickness of the connecting plates. Therefore one of the bolt heads is initially in contact with its respective surface, while the other side representing the nut is displaced by the prespecified amount which would clear the bolt’s hole. In the second step, the contact between the displaced bolt head and its respective surface is activated and the imposed displacement is released, thus forming the desired clamping between the parts via the bolt.

The overall stiffness ‘seen’ by a bolt is affected by several factors, such as the deformation of the connected members, the bolt heads, interaction between the bolts and boundary conditions among others. Therefore it is not possible to use the elastic force–displacement equation of the bolt shaft to determine the displacement needed to induce the desired pretension value in the bolts for the method described above. As a result, a method for inducing accurate pretension values is introduced in this study. In this method, models of a plate with a single headed bolt are separately created to determine the bolt force–displacement relation with different plate thicknesses. The forces at the contact between the bolt head and the plate are computed by displacing the bottom of the bolt shaft. This is a close approximation of the actual pretension force in the connection model to simplify numerical process, which would otherwise require a separate FE analysis for each data point. Each curve is plotted for a certain total thickness of the plates connected by a particular size bolt as shown in Fig. 3. For each bolt size a new group of curves must be created. The advantage of this method is that the bolt geometry is the same as that in the actual connection model.

The calibration curves available are doubly interpolated to extract the correct pre-displacement (bolt shortening) for each bolt in the connection as a function of the total plate thickness and the desired pretension force. The targeted pretension is verified in the actual full connection model. In the case it is not attained, a ‘first order’ correction is imposed on the displacement value as a function of the force difference (pretension error) that has been found. The process is schematically illustrated in Fig. 4.

The experimental data describing the uniaxial stress–strain response is taken from coupon tests performed by Azizinamini [4] and is used to determine the material properties for the FE model. A trilinear stress–strain curve is used in the FE models as shown in Fig. 5, having a modulus of elasticity of 207,218 MPa (30,000 ksi), 276.9 MPa (40.1 ksi) yield stress, and a Poisson’s ratio of 0.3. The bolts are modeled as elastic components in order to ease convergence problems that are occasionally

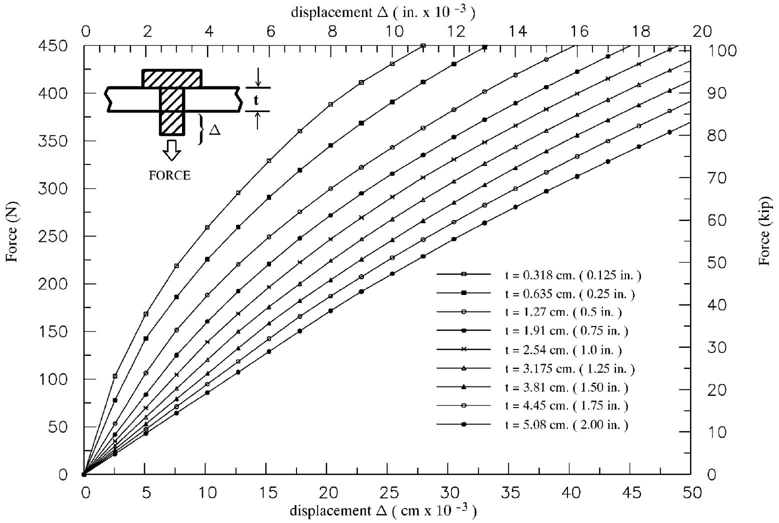

Fig. 3. Bolt pretension calibration curves for 3/4 in diameter bolts used in the FE model.

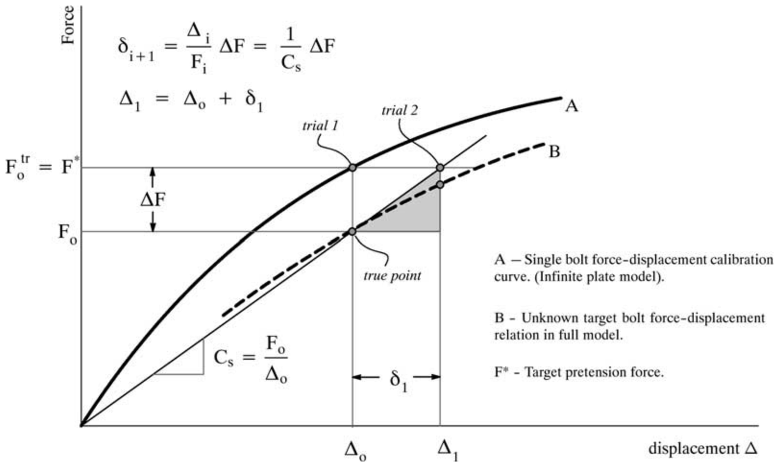

Fig. 4. Schematic representation of the proposed method for calibrating bolt pretension.

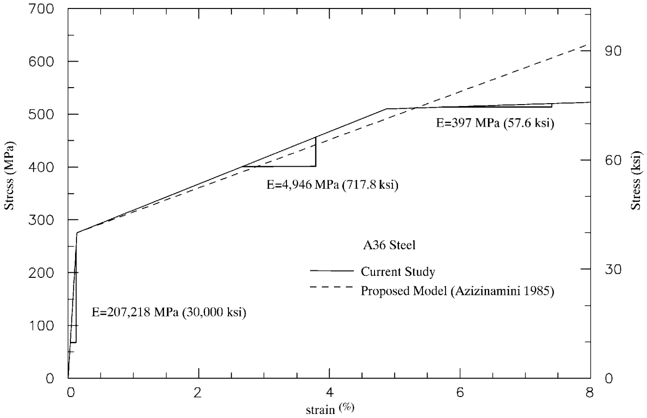

Fig. 5. Uniaxial stress-strain curve.

encountered due to severe localized plastic strain at the corners of the bolt heads. Little or no effect on the overall connection response is observed when compared to results from the models with elastic–plastic bolt material.

The end cross-section of the beam is constrained as a ‘rigid section’. An external displacement is applied. This loading is consistent with what is transmitted to the end of the beam through the pins as shown in Fig. 1. The force–displacement response of the connection is converted to the moment–rotation response using simple relations: $M = F\lambda$, $\phi = \arctan(\Delta/\lambda)$; where M is the moment, ϕ is the rotation of the connection, F is the force, λ is the length of the beam, and Δ is the tip displacement of the beam.

5. Finite element results

5.1. FE and experimental response

The FE analysis results are presented in comparison with the test results and lumped in Fig. 6 for the 14Sx configuration and in Fig. 7 for the 8Sx configuration. Slippage in the connection bolts result in a drop in the moment until bearing is achieved between the bolt shaft and the hole in the plate, as noted by Azizinamini [4] in test 14S2 (Fig. 6) along with an anormality in the initial stiffness in test 8S2 (Fig. 7).

The initial stiffness in general is predicted well by the proposed models. Slippage is observed in the 14Sx configuration tests due to the high moments involved which causes abrupt decreases in the stiffness and reduction of the FE displacement increment size due to slip. The highly non-linear behavior of the connection is related to several factors: friction, bolt pretension, material properties (strain hardening) and connection geometry.

5.2. Effect of friction

It is possible now to extend the study of the connections beyond the experimental data having the prediction capability of the proposed modeling approach. A model is used to study the effect of friction on the response of the connection. Two connection configurations are compared by varying the friction coefficient from 0.25 to 0.5 while keeping the bolt pretension value fixed at 133 kN (30 kips). A friction coefficient value of 0.255 is used instead of 0.25 for the 14S2 test to capture the experimental response. The 14S2 and 8S1 tests are used because they are contrast in terms of beam depth and beam flange angle. The 14S2 connection has a thicker thus stiffer beam flange angle as well a deeper beam which produces more moment in the connection compared to the 8S1 connection.

The variation in the connection response due to the change in the friction coefficient is shown as the shaded region in Figs. 8 and 9. Comparing the two figures, the variation in the response is about 20 times higher in the 14S2 configuration. This shows that friction and slip has more effect on the response of connections with higher moments and stiffer connecting elements.

5.3. Bolt pretension

The pretension value of the bolts has a similar effect, as friction, on the response of the connections. Fig. 10 shows the result of a parametric study using the 14S2 connection while varying the pretension value of the bolts, from 111 kN (25 kips) to 178 kN (40 kips). The friction coefficient is kept fixed at 0.33. Results indicate that the effect of pretension is relatively important and can vary the ultimate moment–rotation by 25 percent.

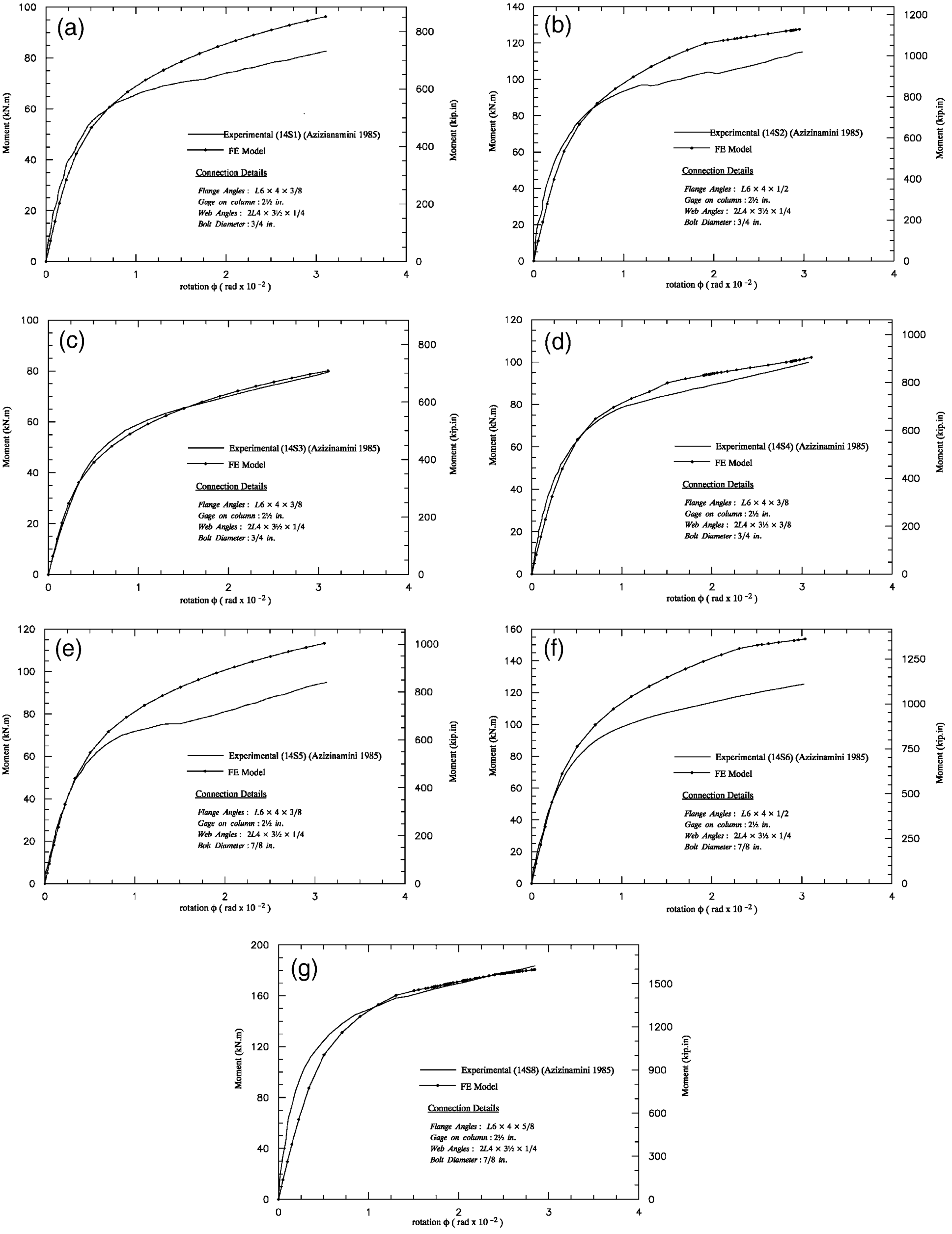

Fig. 6. Moment-rotation curves from experimental and FE models for connections with 14Sx-type configuration.

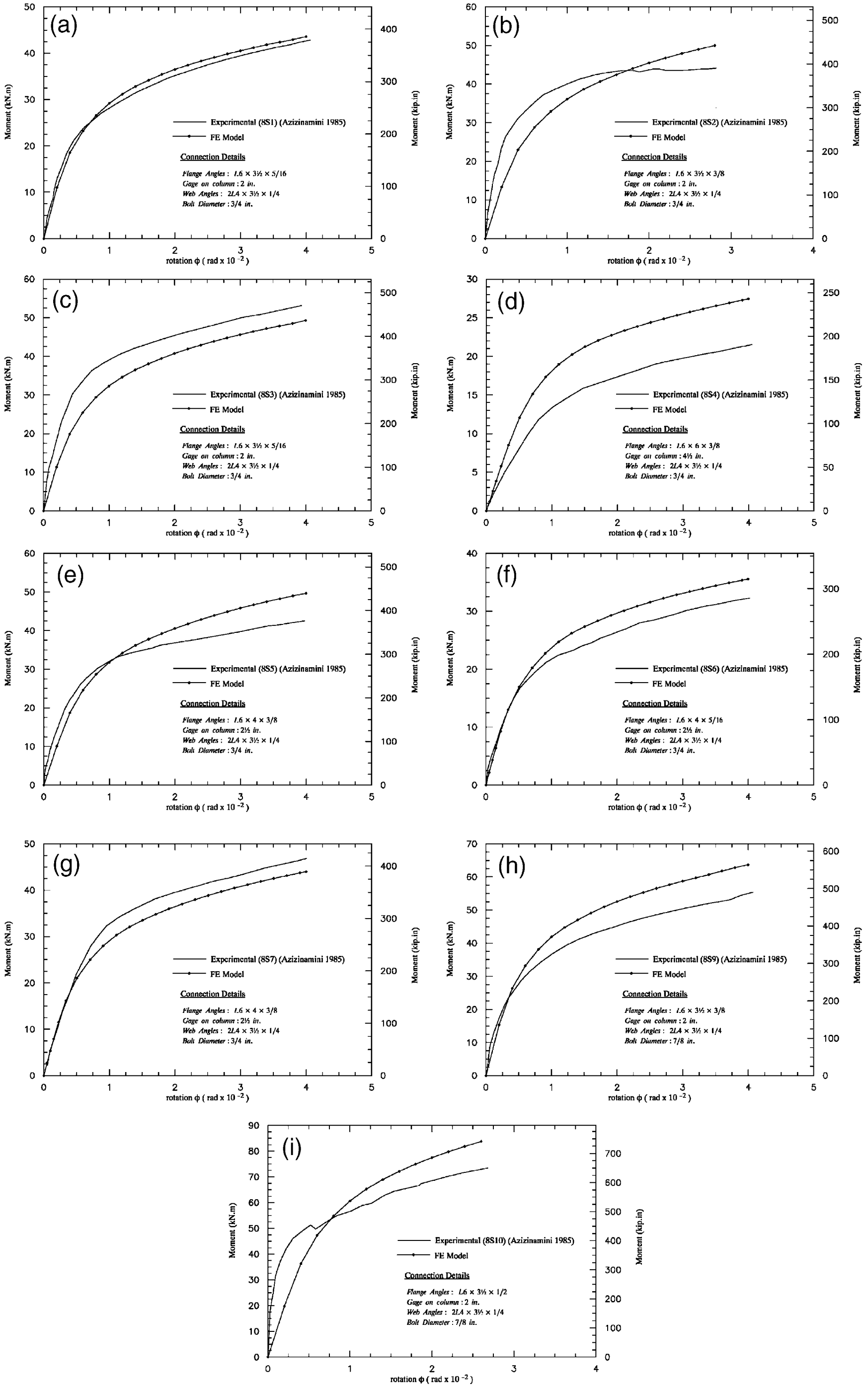

Fig. 7. Moment-rotation curves from experimental and FE models for connections with 8Sx-type configuration

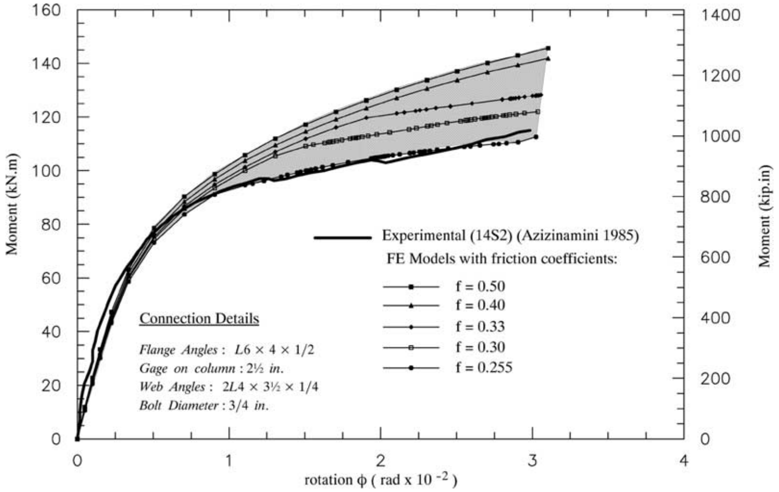

Fig. 8. Moment-rotation response of 14S2 connections with varying friction coefficients.

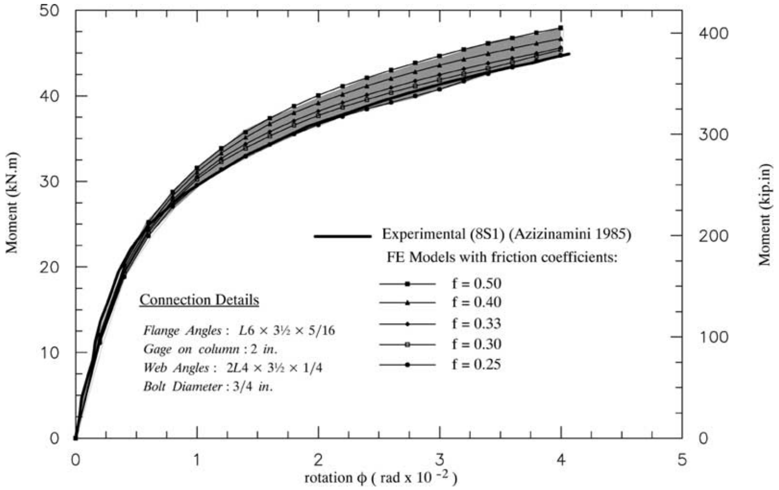

Fig. 9. Moment-rotation response of 8S1 connections with varying contact friction coefficients.

5.4. Pull test approximation

In this section, the ability of a simple pull test to simulate the behavior of top and bottom seat angle connections, with no web angles, is examined. Two tests were conducted by Azizinamini [4] in which the configuration were the same as in tests 14S5 and 14S6 connections, but without the web angles. The FE model and its deformed shape is shown in Fig. 11.

Two pull-test FE models are created. Identical seat angle models of the aforementioned connections are used, with thicknesses of 0.953 cm. (3/8 in.) and 1.27 cm.

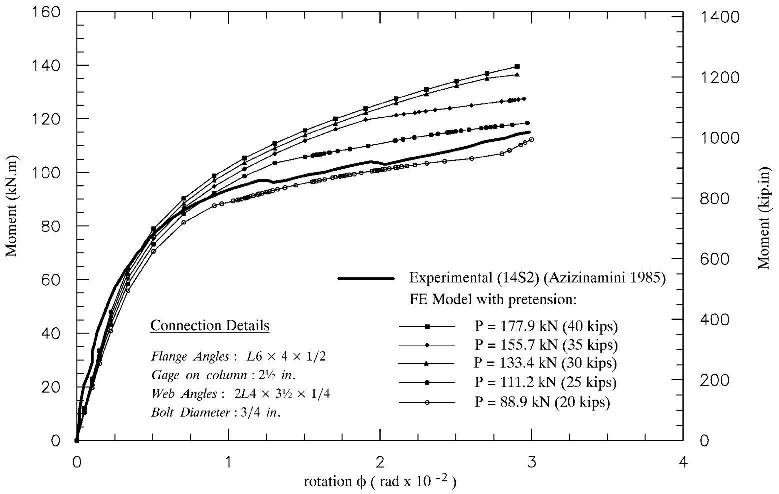

Fig. 10. Moment-rotation response of 14S2 connections with varying bolt pretension.

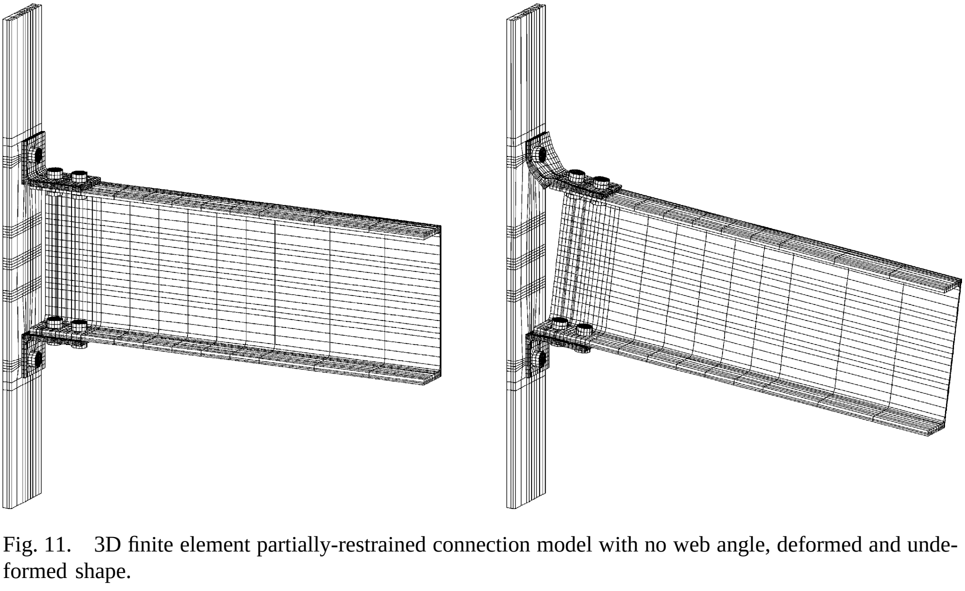

Fig. 11. 3D finite element partially-restrained connection model with no web angle, deformed and undeformed shape.

(1/2 in.). They are assembled as shown in Fig. 12. The force–displacement response of the pull tests are converted to approximate the moment–rotation response of the connection using the relations: $M = Fd$; $\phi =$ arctan (Δ/d); where M is the moment, φ is the rotation of the connection, F is the force, d is the depth of the beam, and Δ is the displacement of the plate representing the beam flange.

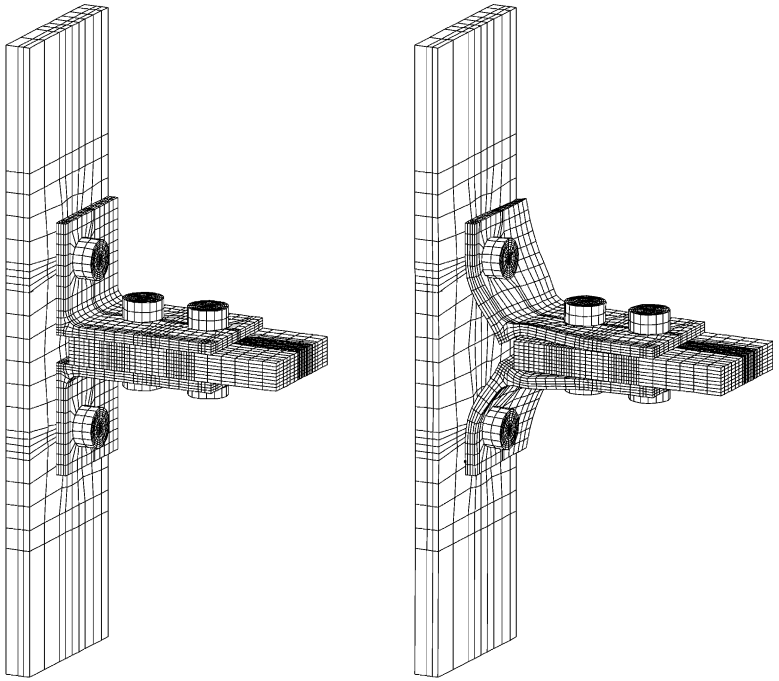

Fig. 12. 3D finite element pull-test model, deformed and undeformed shape.

The converted moment–rotation responses of the pull tests are compared to the responses of the full connections in Figs. 13 and 14. It can be seen that the pull-test model is successful in predicting the initial stiffness of the connection up to a certain point where it begins to diverge from the full connection response. It can be

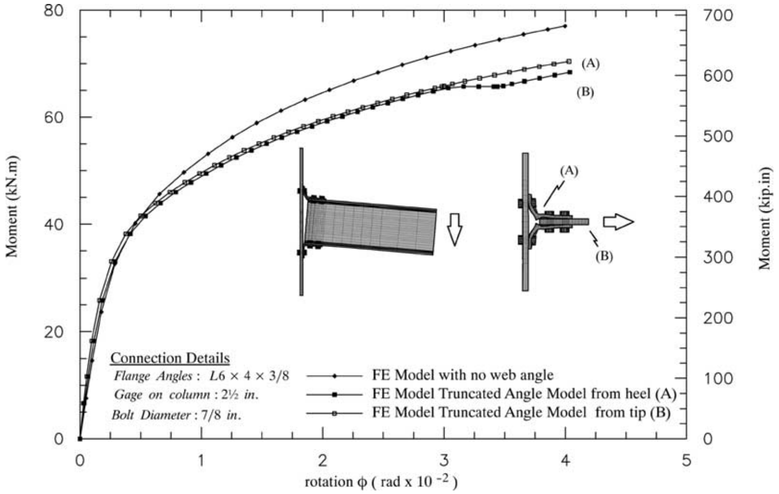

Fig. 13. Moment-rotation curves from pull-test and full-scale models.

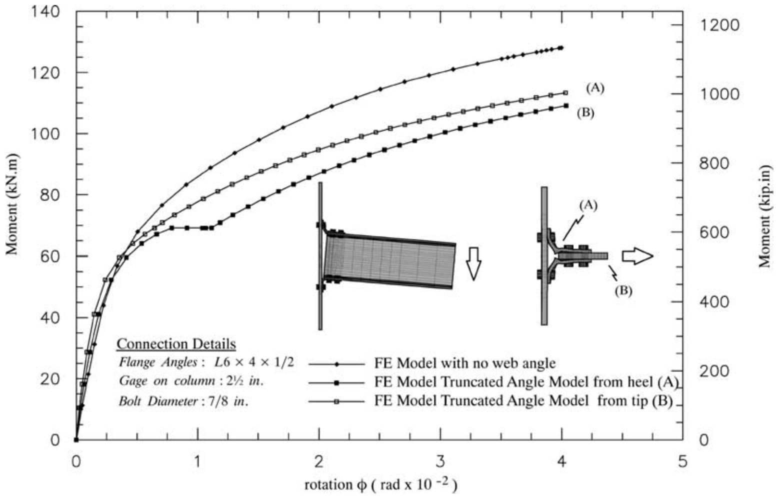

Fig. 14. Moment-rotation curves from pull-test and full-scale models.

seen that the initial stiffness of the connection is governed by the geometry of the top seat angle

The displacement in the pull tests is monitored at both the heel of the angle (A—Figs. 13 and 14) and the tip of the plate pulled (B—Figs 13 and 14). The two curves (A and B) demonstrate the effect of the slip between the angle and the plate. Comparing the two cases, similar to the previous parametric friction study, the slip is more pronounced when the connecting members are thicker (stiffer). The pull tests consistently show a softer response compared to the full model response, especially for higher rotation values.

In order to investigate further the difference between the response of the connection with no web angle and the pull-test response, the plastic equivalent strain fields are compared for the effective top seat angle in Fig. 15. The plastic strain contours at the three different equivalent rotations are shown. The maximum plastic strain values are higher in the pull-test angle until slip occurs, at around 3.0×10^{-2} rad (Fig. 11), where the strains in the connection angle become higher. This may be attributed to the fact that the deformation of the angle in the pull test is limited to a horizontal motion and lacks rotation that is accounted for in the actual connection. The deformation of the angle in the pull test is more constrained than the angle in the full connection. This may add deformation in the angle of the pull test that otherwise does not exist in the full model at higher levels of rotation.

6. Conclusion

A methodology for modeling the moment–rotation response of PR connections is described. Detailed 3D FE models are generated using a parametric library of structural steel shapes. The methodology is efficiently applied for several connections.

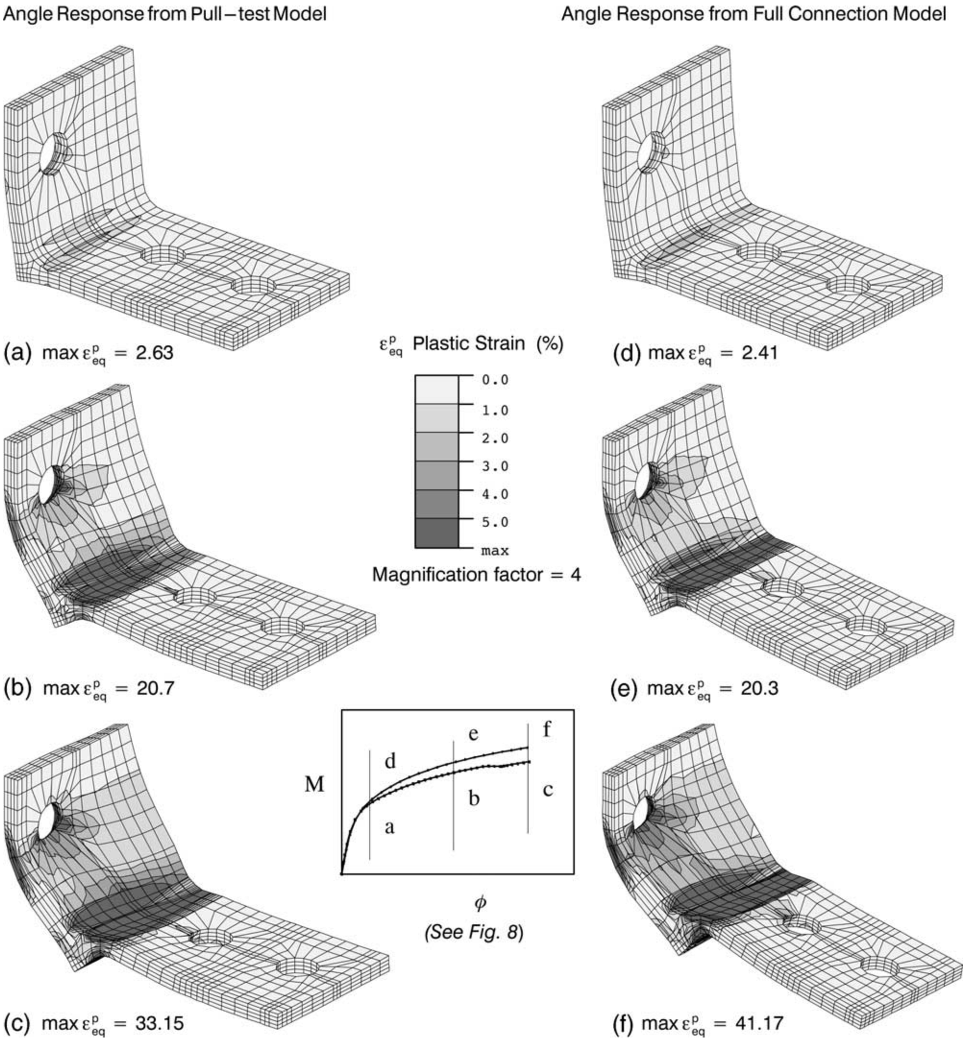

Fig. 15. Comparison of the plastic deformation in the angles from the FE models.

Contact conditions including slip between all the connection components are explicitly recognized. A non-linear incremental plasticity model is used for the steel material. The effectiveness of this modeling approach is extensively demonstrated by comparison with a series of experimental results from a previous study. The inclusion of friction and slip in the model along with the simplicity of changing mesh geometry makes it a general approach for modeling a wide variety of bolted connections.

The force transfer mechanism of clamping the components of the connection with bolts makes it necessary to accurately model the bolt pretension. To this end, a method for pretensioning the bolts in the model and to calibrate their pretension values is introduced. Force–displacement calibration curves, for each bolt size and varying total plate thicknesses, are generated from FE models of a bolt with a semi-infinite plate. These curves are used to extrapolate the amount of displacements to induce the desired pretension levels in the bolts. A simple iterative secant-based scheme is introduced for this purpose.

Having established confidence in the proposed modeling approach by verification with previous experimental work, parametric studies are used to investigate the effect of friction and pretension of the bolts on the connection response. A pronounced effect of friction and slip between the connection components, especially with thicker (stiffer) seat angles, is demonstrated. The approximation of the overall moment–rotation response for top and bottom seat angle connections using pull tests is also investigated.

Refined 3D pull-test models are generated using the proposed modeling approach. Results from the pull tests are compared with 3D full-scale connection models without web angles. The moment–rotations of the pull tests are very close to the full-scale models for relatively small rotations. As rotation increases, a softer response is shown by the pull tests. This may indicate that pull tests are not representative of full-scale models for large rotations.

Acknowledgements

The NSF support under CAREER Grant number 9876080, through the Division of Civil and Mechanical Systems, to the second author is gratefully acknowledged.

References

[1] ABAQUS standard user’s manual. Hibbitt, Karlsson and Sorenson, Inc. 1993.

[2] AISC load resistance factor design specification for steel buildings. 2nd ed. In: Manual of steel construction, (vol. II). Chicago (IL): American Institute of Steel Construction, Inc, 1995.

[3] Azizinamini A. Monotonic response of semi-rigid steel beam to column connections. MS thesis, University of South Carolina, Columbia, 1982.

[4] Azizinamini A. Cyclic characteristics of bolted semi-rigid steel beam to column connections. PhD thesis, University of South Carolina, Columbia, 1985.

[5] Azizinamini A, Radziminski JB. Static and cyclic performance of semi-rigid steel beam-to-column connections. J Struct Eng, ASCE 1989;115(12):2979–99.

[6] Bose B, Sarkar S, Bahrami M. Extended endplate connections: comparison between three-dimensional nonlinear finite-element analysis and full-scale destructive tests. Struct Eng Rev 1996;8(4):315–28.

[7] Bursi OS, Jaspart JP. Basic issues in the finite element simulation of extended end plate connections. Comp Struct 1998;69:361–82.

[8] Chen WF, Goto Y, Liew JYR. Stability design of semi-rigid frames. New York: Wiley, 1996.

[9] Frye JM, Morris GA. Analysis of flexibly connected steel frames. Can J Civ Eng 1975;2:280–91.

[10] Gebbeken N, Rothert H, Binder B. On the numerical analysis of endplate connections. J Constr Steel Res 1994;30:177–96.

[11] Krishnamurthy N. Modeling and prediction of steel bolted connection behavior. Comp Struct 1979;11:75–82.

[12] Kulak GL, Fisher JW, Struik JH. Guide to design criteria for bolted and riveted joints. New York: Wiley, 1987.
[13] Sherbourne AN, Bahaari MR. 3D Simulation of end-plate bolted connections. J Struct Eng, ASCE 1997;120(11):3122–36.
[14] Sherbourne AN, Bahaari MR. Finite element prediction of end plate bolted connection behavior. I: Parametric Study. J Struct Eng, ASCE 1997;123(2):157–64.
[15] TrueGrid Manual. XYZ Scientific Applications, Inc., 1997.
[16] Vasarhelyi DD, Chiang KC. Coefficient of friction in joints of various steel. J Struct Div, ASCE 1967;93(ST4):227–43.
[17] Wanzek T, Gebbeken N. Numerical aspects for the simulation of end plate connections. In: Virdi KS, editor. COST C1: Report of Working Group 6—Numerical simulation of semi-rigid connections by the finite element method. Brussels (Luxembourg), 1999, p. 13–31.
[18] Weynand K, Jaspart JP, Steenhuis M. Economy studies of steel building frames with semi-rigid joints. J Constr Steel Res 1998;46:1–3.
[19] Yang JG, Murray TM, Plaut RH. Three-dimensional finite element analysis of double angle connections under tension and shear. J Constr Steel Res 2000;54:227–44.