New Developments in LS-OPT - Robustness Studies

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Optimization of shell buckling incorporating Karhunen-Loève-based geometrical imperfections

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Layout

- Background and motivation
- Karhunen-Loève expansions for generating random fields
- Non-linear dynamic analysis of buckling
- Optimization set-up
- Cases:
  - Deterministic optimization
  - Stochastic case
  - Optimization of robustness
- Future work

Background and motivation

- Geometrical and material imperfections cannot always be ignored as in deterministic design
- Imperfections are random fields (stochastic processes) and can be modeled
- Shell buckling particularly sensitive to imperfections (both in geometry and in boundary conditions)
- Stochastic analysis (Monte Carlo) required to quantify stochastic variation in non-linear buckling
- For optimal design when geometric imperfections are present, the stochastic analysis has to be incorporated into the optimization process

Karhunen-Loève expansions

- Expand random field

$$\varpi(\mathbf{x},\theta)=\overline{\varpi}(\mathbf{x})+\sum_{i=1}^{\infty}\sqrt{\lambda_i}\,\xi_i(\theta)\,f_i(\mathbf{x})$$

$\overline{\varpi}(\mathbf{x})$ Average random field

f_i eigenfunctions of covariance kernel

λ associated eigenvalues

ξ uncorrelated random numbers

Series representation of a continuous Gaussian process in terms of the spectral expansion of its covariance function

Karhunen-Loève expansions (2)

- Fredholm equation of 2nd kind

$$\int_D C(\boldsymbol{x}_1, \boldsymbol{x}_2) f_i(\boldsymbol{x}_1) d\boldsymbol{x}_1 = \lambda_i f_i(\boldsymbol{x}_2)$$

is solved to obtain f

- Covariance kernel obtained experimentally or specified analytically
  - E.g. Analytical (Exponential)

$$C(x_1, x_2) = \exp\left(\frac{-|x_1 - x_2|}{L_c}\right)$$

Karhunen-Loève expansions (3)

- Solutions to Fredholm equation not available in general, perform numerical integration
- One method is to use Galerkin method
- Basis functions: $f_i(x) = \sum_{k=1}^{N} d_{ik} \phi_k(x)$

Write as error and make orthogonal to basis functions:

$$\int_D \left[\int_D C(x_1, x_2) \sum_{k=1}^{N} d_{ik} \phi_k(x_1) dx_1 - \lambda_i \sum_{k=1}^{N} d_{ik} \phi_k(x_1) \right] \phi_j(x_2) dx_2 = 0$$

Karhunen-Loève expansions (4)

- Eigensystem is obtained

$$\sum_{k=1}^{N} d_{ik}\left[\iint_D C(x_1,x_2)\phi_k(x_1)\phi_j(x_2)dx_1dx_2\right]-\lambda_i\sum_{k=1}^{N} d_{ik}\left[\int_D \phi_k(x_2)\phi_j(x_2)dx_2\right]=0$$

$$AD = \Lambda BD$$

- Efficient to use orthogonal wavelets as basis functions:

So that

$$f_i(x)=\sum_{k=1}^{N} d_{ik}\psi_k(x)=\psi^T(x)D^{(i)}$$

$$C(x_1,x_2)=\sum_{j=1}^{N}\sum_{k=1}^{N}\overline{A}_{jk}\psi_j(x_1)\psi_k(x_2)=\psi^T(x_1)\overline{A}\psi(x_2)$$

where $\overline{A}_{jk}=\frac{1}{h_j h_k}\int_0^1\int_0^1 C(x_1,x_2)\psi_j(x_1)\psi_k(x_2)dx_1dx_2$

Karhunen-Loève expansions (5)

- Solve through 2 successive wavelet transforms for 2D random process
- So that we again get an eigensystem

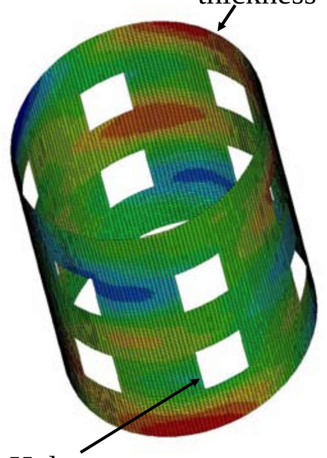
$$\psi^T(x)\overline{A}HD^{(i)} = \lambda_i\psi^T(x)D^{(i)}$$

Or $\hat{A}\hat{D}^{(i)} = \lambda_i\hat{D}^{(i)}$

And finally $f_i(x)=\psi^T(x)H^{-1/2}\hat{D}^{(i)}$

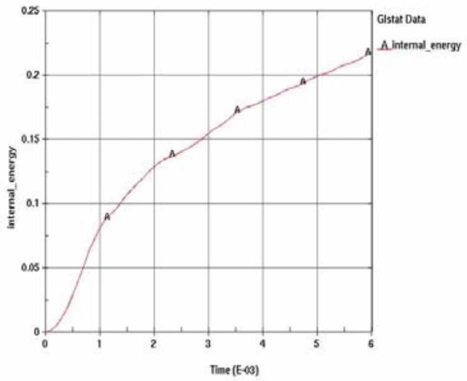
Non-linear dynamic analysis of buckling

- LS-DYNA simulation
- Shell with cut-outs
- Random field of geometrical imperfections superimposed
- Peak normal force and internal energy extracted from simulation
- Parametric model using Truegrid

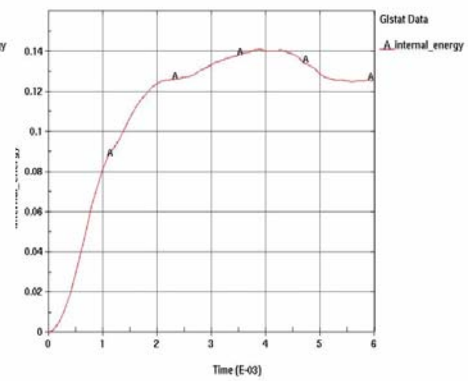


Non-linear dynamic analysis of buckling (2)

- Internal energy



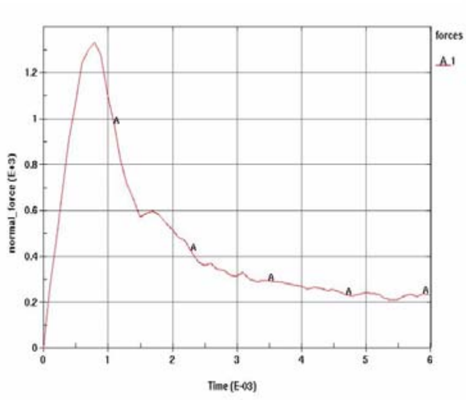
Elastoplastic

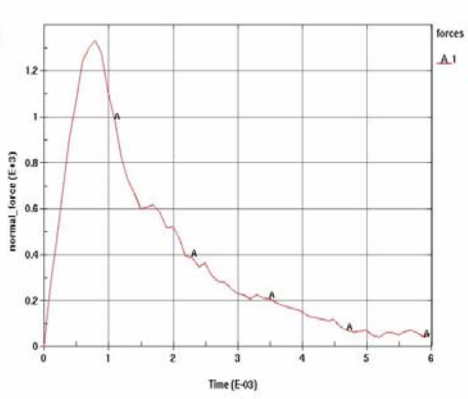


Elastic material model

Non-linear dynamic analysis of buckling (3)

- Peak force

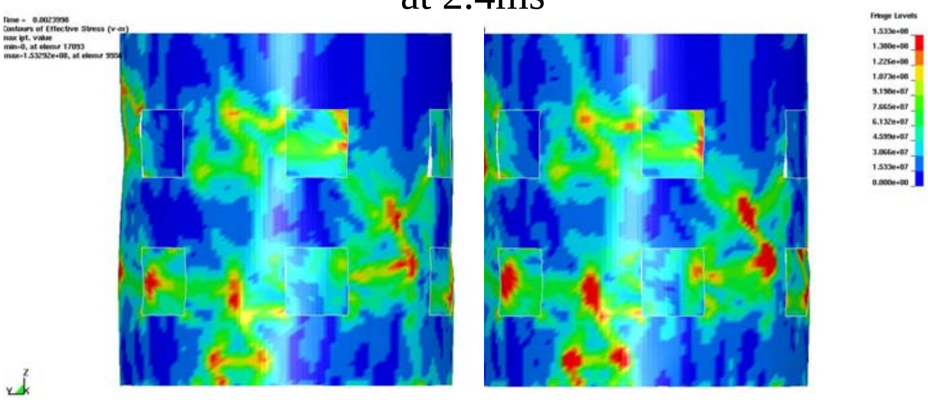




Elastoplastic Elastic material model

Stress on buckled shape

at 2.4ms



Elastoplastic Elastic material model

Optimization set-up

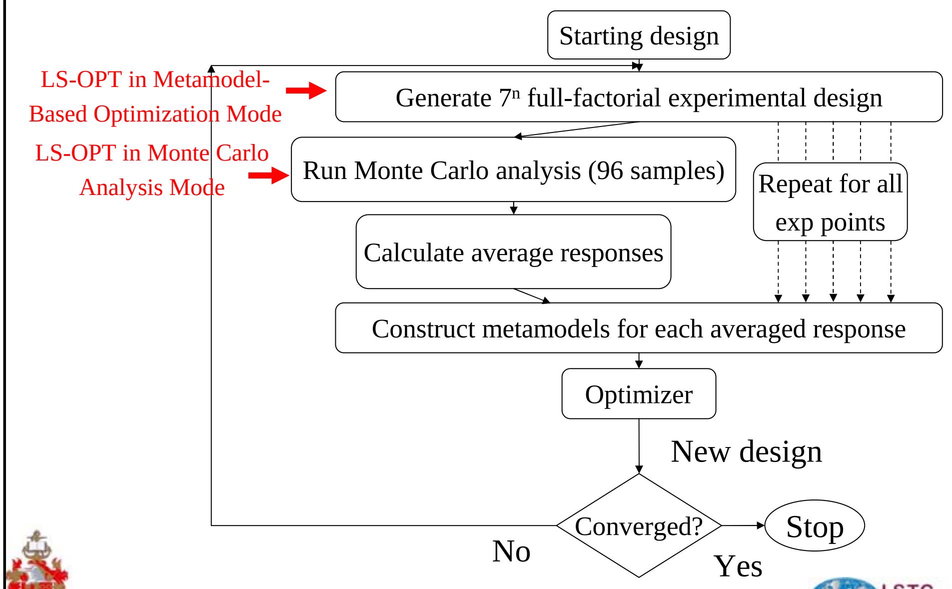
- Design variables: thickness and hole area
- Constraints on maximum peak force and internal energy considered
- Run Monte Carlo analysis at each experimental point to obtain average peak force and internal energy
- Use LS-OPT for Successive Response Surface Method (SRSM) and Monte Carlo analysis

Optimization cases

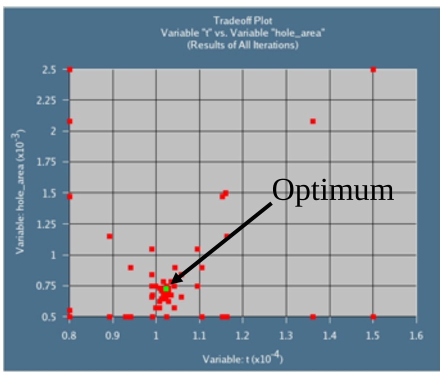
- Case 1: Deterministic optimization
  - Min Mass,
    s.t. Peak Force < 1800, Internal Energy > 0.3
- Case 2: Stochastic optimization
  - Min Mean Mass,
    s.t. Avg Peak Force < 1800,
    Avg Internal Energy > 0.3
- Case 3: Robust optimization
  - Min **COV** (Peak Force),
    s.t. Avg Peak Force < 1800,
    Avg Internal Energy > 0.3

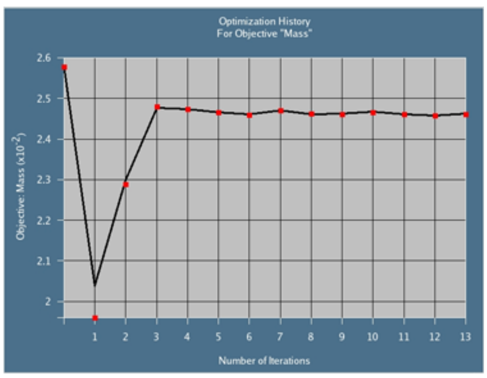
COV
= Coef of Variation
= σ/μ

Optimization flow chart (stochastic)



Case 1: Deterministic optimization



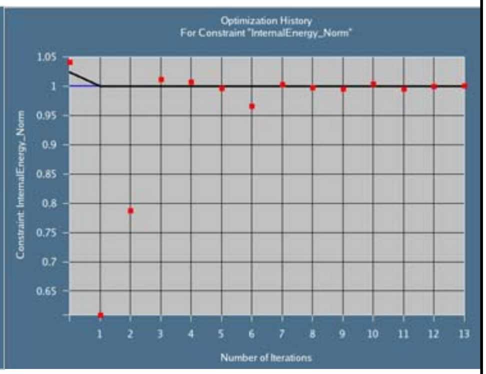


Design variables Objective

Min Mass, s.t. Peak Force < 1800, Internal Energy > 0.3

Case 1: Deterministic optimization (2)

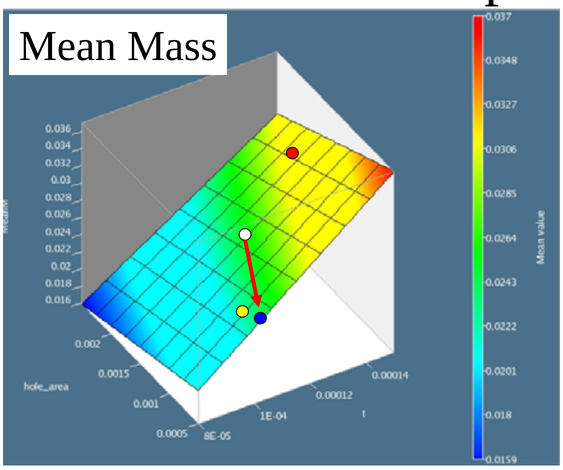
Peak force constraint



Internal energy constraint

Min Mass, s.t. Peak Force < 1800, Internal Energy > 0.3

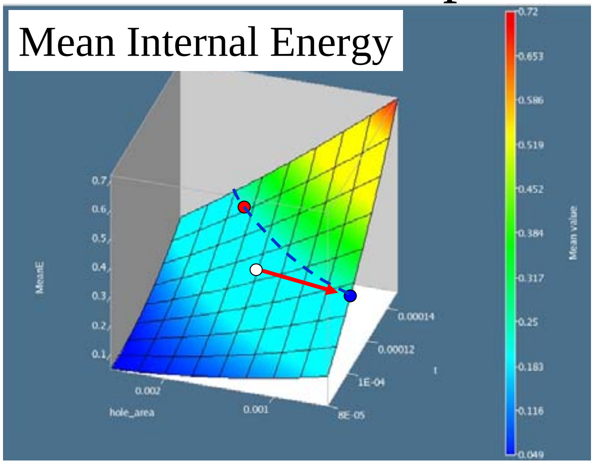
Case 2: Stochastic optimization



- Initial
- Robust
- Stochastic
- Deterministic

Min Mean Mass,
s.t. Avg Peak Force < 1800, Avg Internal Energy > 0.3
Active constraint

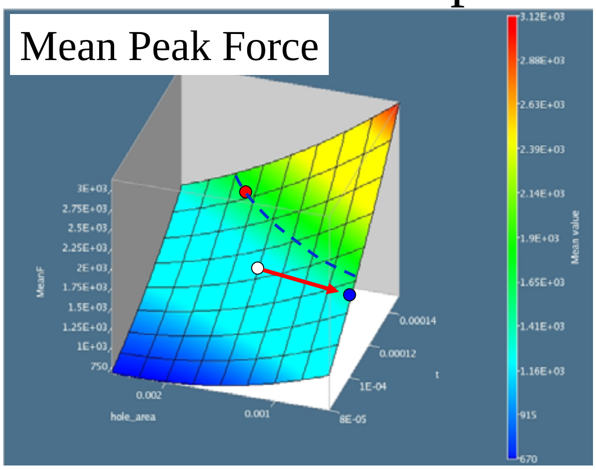
Case 2: Stochastic optimization (2)



- ○ Initial
- ● Robust
- ● Stochastic

Min Mean Mass,
s.t. Avg Peak Force < 1800, Avg Internal Energy > 0.3
Active constraint

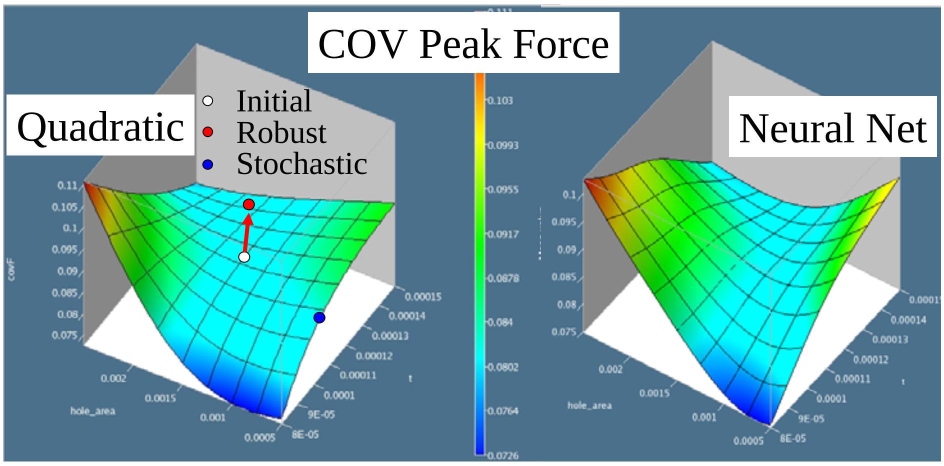
Case 2: Stochastic optimization (3)



- ○ Initial
- ● Robust
- ● Stochastic

Min Mean Mass,
s.t. Avg Peak Force < 1800, Avg Internal Energy > 0.3
Active constraint

Case 3: Robust optimization



Min COV(Peak Force),

s.t. Avg Peak Force < 1800, Avg Internal Energy > 0.3

Active constraint Active constraint

Summary results

Notice different baseline performance

NN fits similar

		Case 1 (Deterministic)		Case 2 (Mean Mass)			Case 3 (COV peak force)		
		Initial	Final	Initial	Quadratic	NN	Initial	Quadratic	NN
Thickness [mm]		0.116	0.102	0.116	0.103	0.105	0.116	0.142	0.140
Hole area [m^2]		0.0015	0.000730	0.0015	0.0005	0.0005	0.0015	0.00182	0.00171
Average mass [kg]		**0.0257**	**0.0246**	**0.0257**	**0.0253**	**0.0259**	0.0257	0.0305	0.0304
Average peak force [N]		1656	1737	1390	1671	1717	1390	1800	1800
Average internal energy		0.313	0.300	0.221	0.3	0.3	0.221	0.3	0.3
Standard deviation	Peak force			124	144	148	124	148	147
	Internal energy			0.0209	0.0129	0.0150	0.0209	0.0318	0.0335
Coefficient of variation	Peak force			0.0892	0.0860	0.0862	**0.0892**	**0.0823**	**0.0815**
	Internal energy			0.0946	0.0430	0.0500	0.0946	0.106	0.112

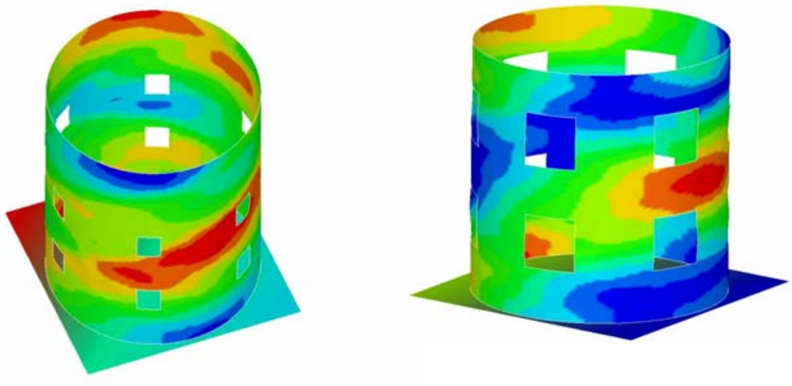
Mass reduced

Similar result (small holes, thin)

COV improved at cost of mass increase (large holes but thick)

Geometrical imperfections of optima

96 random fields for Monte Carlo runs



Stochastic optimum Robust optimum

Conclusions

- Feasible, but expensive, to include stochastic and robustness effects into optimization process
- Inclusion of stochastic effects did not modify deterministic optimum significantly
- Robust optimization led to a much heavier design
- Robust and stochastic optimization process fully automated using LS-OPT in Metamodel mode to call LS-OPT in Monte Carlo Analysis mode

Future work

- Extend simulation to consider post-buckling performance
- Apply to more realistic examples (require measured geometric imperfections)

Post-buckling